P425/1
PURE MATHEMATICS
July/august 2024
3 HOURS

JOURNEY OF SUCCESS EXAMINATIONS BOARD

Uganda advanced certificate of education

MOCK EXAMINATIONS
PURE MATHEMATICS
Paper 1
3 HOURS

INSTRUCTIONS TO CANDIDATES

- *Answer **all the eight** questions in section **A** and any **five** from section **B**.*
- *Any additional question(s) will **not** be marked.*
- *All working **must** be shown clearly.*
- *Begin each question on a fresh sheet of paper.*
- *Silent, non-programmable scientific calculators and mathematical tables with a list of formulae may be used.*

TURN OVER

SECTION A (40 MARKS)

Answer all questions in this section

1. Show that (x – 3) is a factor of $x^3 - 19x + 30 = 0$. Hence determine the other roots.

(05 marks)

2. Determine the equation of the normal to the parabola with focus **S**(3,0)at the point (3,6)

(05 marks)

3. Find the values of k for which the following equations have equal roots.
   a) $3x^2 + kx + 12 = 0$
   b) $x^2 - 5x + k = 0$ **(05 marks)**

4. Given the quadratic equation $ax^2 + bx + c = 0$, show that roots of x are given by

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$ **(05 marks)**

5. If the surface area ***S*** of the cylinder($S = 2\pi r^2 + 2\pi rh$) is kept constant, show that the volume ($V = \pi r^2 h$)of the cylinder will be maximum when $h = 2r$.

(05 marks)

6. Find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ in terms of t in the expressions $x = t^2$ and $y = 4t - 1$ **(05 marks)**

7. Sketch the circle $x^2 + y^2 + 2x - 4y - 4 < 0$ and show the required region. **(05 marks)**

8. The sum of the three terms in A.P is 30 and the sum of their squares is 398. Find the numbers. **(05 marks)**

SECTION B (60 MARKS)

Answer any five questions from this section. All questions carry equal marks

9 a) Prove that for all integral values of n, $(\cos\theta + i\sin\theta)^n = (\cos n\theta + i\sin n\theta)$.

Hence find the value of $(\cos\frac{1}{4}\pi + i\sin\frac{1}{4}\pi)^{12}$ **(06 marks)**

b) Use De-moivre's theorem to show that; $tan3A = \frac{3tanA - tan^3A}{1 - 3tan^2A}$ **(07 marks)**

10 a) Solve the equation **4tan** θ **tan 2**θ **= 1** from $0 \leq \theta \leq 360°$ **(04 marks)**

b) Prove that; $tanA + \tan B = \frac{\sin(A+B)}{cosAcosB}$ **(03 marks)**

c) Without using tables or calculator, evaluate tan 195° **(03 marks)**

TURN OVER

11 *a)* Find the area enclosed by the x-axis, the curve $y = 3x^2 + 2$ and the straight lines $x = 3$ and $x = 5$. **(04 marks)**

b) Find $\int \frac{\cos 2x}{\sin^2 2x} dx$ **(04 marks)**

c) Express $f(x) = \frac{5}{(x-2)(x+3)}$ into partial fractions. Hence $\int f(x)dx$ from 0 up to 2 **(04 marks)**

12 a) When a polynomial $p(x)$is divided by x–1, the remainder is 5 and when $p(x)$ is divided by x–2, the remainder is 7. Find the remainder when the same expression is divided by (x-1)(x-2) **(06 marks)**

b) The roots of the equation $2x^2 + 7x - 3 = 0$ are α and β. Find the equation whose roots are $\left(\propto + \frac{5}{\beta}\right)$ and $(\beta + \frac{5}{\propto})$ **(06 marks)**

13 a) Use Pascal's triangle to expand $(x - \frac{1}{x})^5$ **(04 marks)**

b) Use maclaurin's theorem to expand $\sqrt[3]{(1 + X)}$, up to x^3. Hence find the approximate values for $\sqrt[3]{(1.03)}$ correct to four places of decimals. **(08 marks)**

14 a) Find the equation of a plane passing through a point A with a position vector $-2\boldsymbol{i} + 4\boldsymbol{k}$ and is perpendicular to the vector $\boldsymbol{i} + 3\boldsymbol{j} - 2\boldsymbol{k}$. **(04 marks)** b) Find the line of intersection of the two planes $4x + 6y + 8z = 2$ and $x + y + 3z=0$. **(04 marks)**

c) Find the Cartesian equation of a plane passing through A,(1, 1, 1), B(5,0,0) *and C*(3, 2,1) **(04 marks)**

15 a) Prove that the circles $x^2+y^2+4x-2y-11=0$ and a circle whose radius is 3 and centre C(2,4) are orthogonal**.** **(06 marks)**

b) Given the parabola $y^2 = 4ax$, determine the point of intersection of the two tangents at P(4,8) and at point Q(6 ,$\frac{9}{4}$) **(06 marks)**

16 a) solve the differential $\boldsymbol{x^2 \frac{dy}{dx} + 2xy = 1}$ **(04 marks)**

b) The rate at which the temperature of a body reduces is proportional to the amount by which its temperature exceeds that of its surroundings. If the temperature of an object falls from 200° to 100° in 40 minutes, in a surrounding temperature of 10°.

i) Prove that after *t minutes*, the temperature, *T* degrees, of the body is given by $T=10+190e^{-kt}$, where $k = \frac{1}{40} In(\frac{19}{9})$

ii) Calculate the time it takes to reach 40°. **(08 marks)**

END